RAN-2203000205023002

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T.Y.B.Sc (Mathematics) (Sem. V) Examination October - 2023

MTH-502 - Linear Algebra - I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc (Mathematics) (Sem. V)**

Name of the Subject :
☛ **MTH-502 - Linear Algebra - I**

Subject Code No.: **2203000205023002**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q. 1 Answer the following. (Any Five) **(10)**

(1) Let $f : (x, y) \to x * y = y$.

Is the binary operation $*$ be an associative and commutative? Justify your answer.

(2) In a vector space R^+, for every $u, v \in R^+$ if $\alpha u = u^\alpha$ then find Identity and Inverse element with respect to the operation multiplication.

(3) Give an example to show that $(U \cup W)$ is not in general subspace.

(4) Is the sum (XY – plane + Z – axis) = V_3 a Direct sum? Justify your answer.

(5) Find the dimension of [(1,2,3), (3,1,0), (–2,1,3)].

(6) Is a basis never include the zero vector? Justify your answer.

(7) prove that any set containing two L.I vectors in a vector space V_2 is basis of it.

(8) In V_3 ; [Y – axis $\cup$ Z - axis] = V_3 is true ? Justify your answer.

Q. 2 **Answer the following. (Any two)** **(10)**

(1) Let R^+ be the set of positive real numbers. Define the operations addition and scalar multiplication as follows.

$u + v = u.v$ for all $u, v \in R^+$ and $\alpha u = u^\alpha$ for all $u \in R^+$ and real scalar α.

Prove that R^+ be a commutative group with respect to addition.

(2) Let W be the set of all vectors of the form $x(1,2,3)$; $x \in R$ in V_3. Then prove that W is a subspace of V_3.

(3) Prove that the set $\{(x_1, x_2, x_3) \in V_3 / x_1 = \sqrt{2}\, x_2$ and $x_3 = 3x_2\}$ is a subspace of V_3.

Q. 3 **Answer the following (Any two).** **(10)**

(1) If S is a non empty subset of a vector space V, then prove that $[S] = S$ if and only if S is a subspace of V.

(2) Let U and W be two subspace of vector space V then prove that $U + W = U$ if and only if $W \subset$ U.

(3) Let S be a non empty subset of a vector space V and $u, v \in V$. If $u \in [S \cup \{v\}]$ but $u \notin [S]$ then prove that $v \in [S \cup \{u\}]$.

Q. 4 **Answer the following. (Any two)** **(10)**

(1) Prove that : If a set is L. I then any subset of it also L. I.

(2) (a) In a vector space V, prove that if v is a linear combination of $v_1, v_2, \ldots \ldots \ldots, v_n$

V then prove that the set $\{v, v_1, v_2, \ldots \ldots \ldots, v_n\}$ is L.D.

(b) Prove that : If v_1, v_2, v_3 are coplanar then the set $\{v_1, v_2, v_3\}$ is L.D.

(3) Show that the ordered set $S_4 = \{(1,1,0), (0,1,1), (1,0,-1), (1,1,1)\}$ is L.D and locate one of the vectors belongs to the span of the previous ones. Also find largest L.I subset A of S_4 with condition $[A] = [S_4]$.

Q. 5 **Answer the following (Any two).** **(10)**

(1) Verify the following results by giving one illustration to each:

(i) If U and W are subspaces of a finite dimensional vector space V then $\dim(U + W) = \dim U + \dim W - \dim (U \cap W)$.

(ii) If U and W are subspaces of a finite dimensional vector space V such that $U \cap W = \{0\}$ then $\dim(U \oplus W) = \dim U + \dim W$.

(2) In a vector space V, the set $B = \{v_1, v_2, \ldots \ldots, v_n]$ generates V. Then prove that the expression $v = \alpha_1 v_1 + \alpha_2 v_2 + - - - - + \alpha_n v_n$ is unique for every $v \in V$ then prove that the set B linearly independent set.

(3) Find the general form of a co-ordinate vector of a vector $(-\sqrt{2}, \pi, e)$ relative to the ordered basis $\{(2,1,0), (2,1,1), (2,2,1)\}$.

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